Quantum Circuit Simplification and Extraction

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Overview of Quantum Computing

Overview of Quantum Computing

Quantum Computing is centered on developing computer technology based on quantum theory.

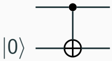
Using the model of binary circuits, we can define:

- Qubits - a complex vector which stores spin and polarization.
- Gates - a unitary matrix which can be applied to our qubits.

We use these parts to form the basis for quantum circuits, which allow us to interface with a quantum computer.

Quantum Circuits are made up of a number of qubits and a collection of gates, which perform operations in the circuit.

Table 1: Common Quantum Gates and Corresponding Matrices

Pauli Gates		$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0 & -i\\i & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$
Hadamard Gate		$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$
Controlled Not (CNOT)		$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}$

Quantum Circuit Example

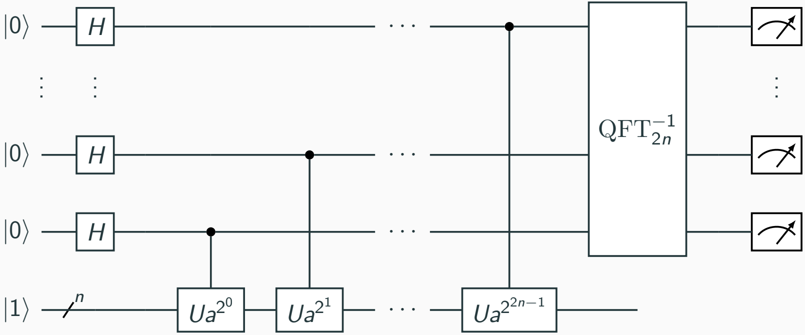


Figure 1: Shor's Algorithm[1]

[1]Beauregard, "Circuit for Shor's algorithm using 2n+3 qubits"; Shor, "Algorithms for quantum computation: discrete logarithms and factoring".

Circuit Simplification

Similar to classical computers, quantum circuits must be compiled to a language understood by the quantum computer.

Each operation in a quantum circuit is prone to error from disturbances in their environment, which can be an issue for larger circuits with many gates.

We can use techniques such as simplification to reduce the number of gates in larger circuits without changing the nature of the circuit.

The Language of the ZX-Calculus

The ZX-Calculus

The ZX-Calculus, a graphical language introduced by Coecke and Duncan[2], allows us to represent a quantum circuit as a graph, called ZX-diagrams.

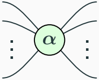
A ZX-diagram is a graphical representation of a quantum circuit, which come equipped with rewrite rules that allow us to perform graph operations.

The ZX-calculus is built from red and green spiders[3]

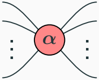
[2] Coecke and Duncan, “Interacting Quantum Observables”, “Interacting quantum observables: categorical algebra and diagrammatics”.

[3] If you have difficulty distinguishing green and red, Z spiders will appear lightly-shaded and X spiders darkly-shaded.

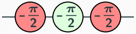
ZX-Calculus Generators

Z-spider: 

$:= |0 \cdots 0\rangle\langle 0 \cdots 0| + e^{i\alpha} |1 \cdots 1\rangle\langle 1 \cdots 1|$

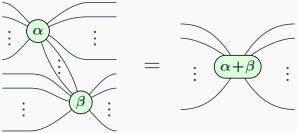
X-spider: 

$:= |+ \cdots +\rangle\langle + \cdots +| + e^{i\alpha} |- \cdots -\rangle\langle - \cdots -|$

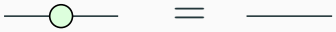
Hadamard:  $:=$ 

Rewrite Rules

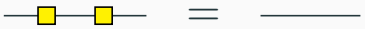
Spider Fusion



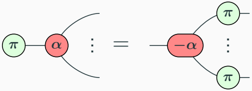
Identity Removal



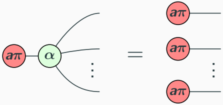
Hadamard Cancellation



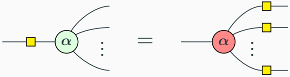
π commutation



State Copy



Color Change



Rewrite Example

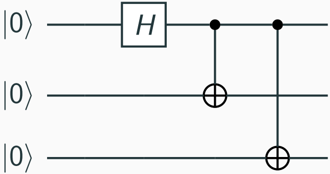


Figure 2: GHZ State

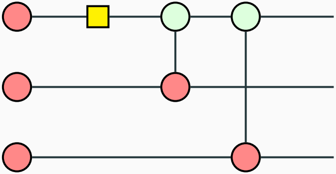


Figure 3: ZX-Diagram

Graph-theoretic Simplification

A ZX-Diagram is considered **clifford** if all phases in the diagram are multiplies of $\frac{\pi}{2}$.

Using the definition[3] for a *graph-like* ZX-diagram, we can convert our ZX-Diagram into this form, as shown below:

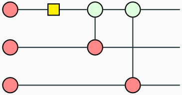


Figure 4: ZX-Diagram of GHZ State

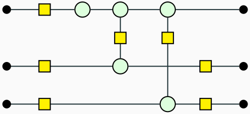


Figure 5: Graph-like ZX-Diagram

[3] Duncan et al., "Graph-theoretic Simplification of Quantum Circuits with the ZX-calculus", Definition 3.1.

Simplification Strategy

In ZX-Diagrams we call a spider *interior* when it is not connected to an input or output, otherwise it is called a *boundary* spider.

Using two new rewrite rules, we can form a routine to remove as many *interior* spiders as possible.

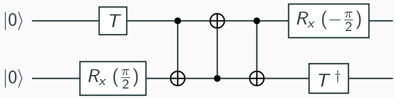


Figure 6: Quantum Circuit

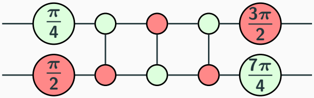
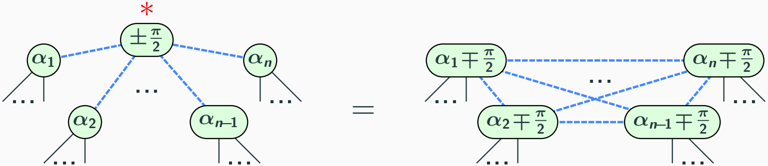
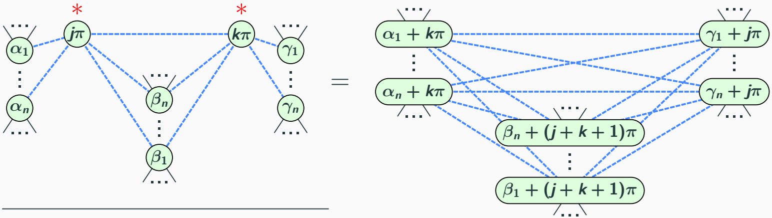


Figure 7: ZX-Diagram of Circuit

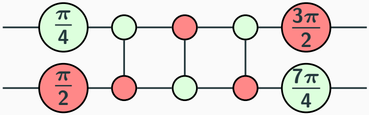
Local Complementation[4]



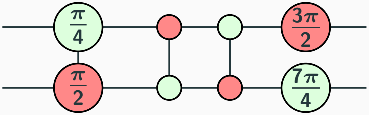
Pivot Simplification[4]



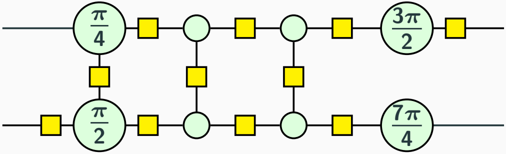
[4]Duncan et al., "Graph-theoretic Simplification of Quantum Circuits with the ZX-calculus", Section 4.



ZX-Diagram

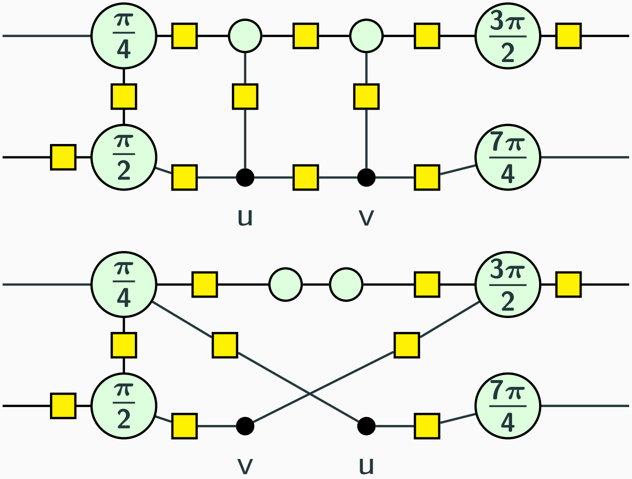


Spider Fusion



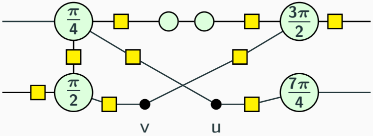
Conversion to Graph-like ZX-Diagram

Simplification Example

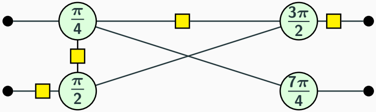


Pivot Simplification

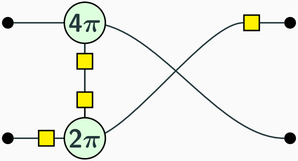
Simplification Example



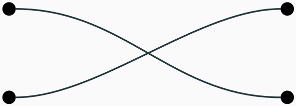
Pivot Simplification



Spider Fusion



Identity Removal



Final Simplification

Circuit Extraction

Circuit Extraction

In order to use our simplified ZX-Diagram on a quantum computer, we must convert our graph representation back to a quantum circuit.

Conversion from our ZX-Diagram to a quantum circuit is simple, provided the simplified graph does not contain any interior spiders[5].

Unfortunately, while the rewrite rules for ZX-diagrams allow us to simplify circuits efficiently, it does not currently guarantee extraction for non-Clifford diagrams.

[5] Duncan et al., "Graph-theoretic Simplification of Quantum Circuits with the ZX-calculus", Section 6, 7.

Conclusion

Using the powerful language of the ZX-Calculus we form rules and routines to simplify a quantum circuit and extract it, further progressing the ability of quantum computing.

PyZX[6] is a brilliant tool for working with ZX-Diagrams in Python and implements all the rules and methods shown in this presentation.

This field of Quantum Computing is very active with new research giving new methods for further simplification and extraction techniques.

[6] Kissinger and Wetering, "PyZX: Large Scale Automated Diagrammatic Reasoning".

Thank you for listening!

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